Unit - 1: Relations and Functions

Values and Attitudes: Respect others, Tolerance, relationship values

Time: 9 Periods

Concepts/ Ideas	Process/Activity with assessment	Learning outcome
1. Basic idea of Relations.	**General discussion** **Discussion points** • Idea of relations with suitable examples for recalling the pre-requisites. **Assessment** ▪ Find the domain, co-domain and range of different relations	• Recalls the idea of relations.
2. Empty relation and universal relation on a set (trivial relations)	**General discussion** **Discussion points** • Different examples for a relation defined on a set to get the concept of empty relation, and universal realation on A. **Assessment** ▪ Find the trivial relation from a group of relations.	• Identifies and classifies trivial relations.
3. Reflexive, symmetric, transitive and equivalence relations.	**General discussion** **Discussion points** • Suitable examples to get the idea of reflexive, symmetric, transitive and equivalence relations. **Assessment** ▪ Check whether the given relations are equivalence or not.	• Classifies and explains equivalence relations. • Illustrates different equivalence relations.
4. Equivalence class	**Group activity** To find the set of all elements related to each elements in a set for a given equialence relation. • Equivalence class • Partition of a set. **Assessment** ▪ Find the equivalence classes of different elements in a set for a given equivalence relations	• Classifies different classes and identifies their properties. • Finds the partition of a set using an equivalence relation.

Concepts/ Ideas	Process/Activity with assessment	Learning outcome
5. Basic ideas of functions.	• ICT based discussion about the basic ideas of functions. **Assessment** ▪ Draw the graph of the given function. ▪ Find the domain and range of the function.	• Recalls the idea of functions. • Finds their domain and range
6. Types of functions. • one-one (injective) • on to (surjective) • Bijective functions	• **Lab work and ICT presentation**. Use arrow diagrams, and graphs of different functions to get the conepts. • Inverse of a function exists if and only if it is bijective. **Assessment** ▪ Classify the given functions into one-one, onto and bjective. ▪ Redefine domain and co-domain of a function to make it as a bijective function.	• Identifies the need of bijective function. • Examines different types of functions. • Interprets and modifies the domain and co-domain of a function so that the function is one-one and onto.
7. Composition of functions	**General discussion** **Discussion points** • Use arrow diagram of functions. **Assessment** ▪ Find the composition of given functions.	• Illustrates composition of functions. • Constructs new functions using composition. • Idenfies composition as an operation
8. Invertible functions.	• General discussion and ICT presentaiton find the condition for invertible functions. **Assessment** ▪ Find the inverse of a given function.	• Identifies the conditions for invertible functions. • Applies the concept of composition of functions to find the inverse of a function.

Concepts/ Ideas	Process/Activity with assessment	Learning outcome
9. Binary operations and their properties.	**General discussion** **Discussion points** • Discuss the operations ('+', '–', '×', ÷, ∪, ∩, *fog* etc) and their properties. • Discuss the binary operations and their properties on a set.	• Identifies the binary operation as a function. • Illustrates various properties of a binary operation. • Identifies the identity element of a binary operation, the inverse element of an element belongs to the set.

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Unit: I . Relations and Functions

Values and Attititudes:

Overview

In plus one class students are familiar with the concepts of Sets, Relations, functions and their domain and range. Sets are well-defined collection of objects, a relation between A and B indicates relationships between members of the sets A and B and functions are a special type of relation where there is at most one relationship for each element a ∈A with an element in B. The Cartesian product allows a new set to be created from existing sets. The Cartesian product of two sets A and B (denoted AX B) is the set of ordered pairs {(a,b)|a ∈ A, b ∈ B}. A relation R is a subset of the Cartesian product (A × B) of A and B where A and B are non-empty sets. The domain of the relation is A and the co-domain of the relation is B. The notation ***aRb*** signifies that there is a relation between a and b and that (a, b) ∈ R.

A function f : A→B is a special relation such that for each element a ∈A there is exactly one element b ∈ B. This is written as f (a) = b. A function is a relation but not every relation is a function. In this chapter we deal with equivalence relations and equivalence classes , types of functions such as injective, surjective, bijective and the operation- composition of functions and inverse of a function. Finally we discuss binary operation as a function and its properties.

Through the unit

Concept: Types of functions- One-one , Onto and Bijective functions

Activity: Lab Work and ICT Presentation,

Learning Outcome:

  - Identifies the need of bijective function.
- Examines different types of functions.
  - Interprets and modifies the domain and co-domain of a function so that the function is one-one and onto.

Principle: A relation from A to B is a function if every elements of A have a unique image in B

Procedure: Draw the arrow diagrams of the following functions.

1. Let f_1 :A→B, where A={1,2,3,4} and B={3,4,5,6} defined by

   f_1= {(1,3),(2,5), (3,4),(4,3)}

2. Let f_2 :C→D, where C={1,2,3} and D={3,4,5,6} defined by

   f_2= {(1,3),(2,4), (3,5)}

3. Let f_3 :E→F, where E={1,2,3,4} and F={7,8} defined by
   f_3= {(1,7),(2,8), (3,7),(4,8)}
4. Let f_4 :G→H, where G={1,2,3} and H={4,5,6} defined by f_4= {(1,4),(2,6), (3,5)}

Draw the arrow diagrams of the following relations and check whether they are functions or not.

1. g_1:{(y,x): y∈B,x∈A and y=f_1(x)}
2. g_2:{(y,x): y∈D, x∈C and y=f_2(x)}
3. g_3:{(y,x): y∈F, x∈E and y=f_3(x)}
4. g_4:{(y,x): y∈H, x∈G and y=f_4(x)}

Discussion points:

1. Conditions for g_1, g_2 , g_3 and g_4 to be a function
2. The conditions that the different elements have different images lead the concept of the One-One function and every element in the co-domain has a pre-image in the domain leads to the concept of onto function.
3. Conditions of bijection follows from above discussions.
4. Inverse of a function exists if and only if it is bijective.

Assessment: Draw the graphs of the following functions.

- Let f:R→R defined by f(x)=3x+2

- Let f:R→R defined by f(x)= x^2
- Let f:R→R defined by f(x)= sin x

Check whether the above functions are bijecive? If not, redefine the domain and co-domain of the function to make it bijective.

Evaluation

Choose the correct answer(1-5)

1. The relation R = { (1,1),(2,2),(3,3)} on the set A = { 1,2,3 } is
   a) Symmetric only b) reflexive only c) transitive only d) an equivalence relation
2. If f(x) = $\frac{2x+1}{3x-2}$, then (fof)(2) =
   a) 1 b) 2 c) 3 d) 4
3. The function f : R → B defined by f(x) = x^2+1 is an onto function, then B =
   a) R b) [0, ∞) c) [-1,1] d) [1, ∞)
4. Let f : (4,6) → (6,8) be function defined by f(x) = x+ $\left[\frac{x}{2}\right]$, (where [.] denotes the greatest integer function), then $f^{-1}(x)$ =
   a) x-$\left[\frac{x}{2}\right]$ b) -x-2 c) x-2 d) 2-x
5. In set of non zero real numbers, the binary operation * is defined by a*b = ab/9, then the inverse of 81 is
   a) 1 b) 81 c) 3 d) 9
6. Let R be a relation on the set of Integers given by R = { (a,b) : a = 2^kb, for some integer k}. Check whether R is an Equivalence relation or not?
7. Let f : R → R be a function defined by f(x) = x^2. Redefine the domain and co-domain of the function so that the given function is invertible. Hence find its inverse function.
8. Let f,g : R → R be functions defined by f(x) = 2x+3 and g(x) = 3x+2.
   Verify that $(gof)^{-1} = f^{-1}og^{-1}$
9. Let ∗ be a binary operation defined on the set of real numbers by a∗b=1+ab. Check whether ∗ is commutative or associative.
10. Let ∗ be a binary operation defined on A=Z x Z, by (a,b) ∗(c,d) = (a+c, b+d). Find the identity element for ∗ on A if any. Also find the inverse element of (a,b) ∈ A if exixts.

Answers

1. d 2. b 3. d 4 . c 5. a

6. It is an equivalence relation

7. We can choose any suitable domain and co-domain to make the function bijective.
For Eg: If domain=[0,∞) and Co-domain = [0,∞) . It is bijective. Inverse function is x=√y
9. ∗ is commutative but not associative.

10 . Identity element is (0,0) Invese of (a,b) is (-a,-b)

CE Activity

Assignment: Let R be a relation defined on NXN by *(a,b)R(c,d) if and only if ad=bc.* Prove that R is an equivalence relation. Find the set of all elements related to the element (2,3).Also find the equivalent class of (a,b)∈NXN.

Unit Assessment:

Class Test : Include questions like the following

1. Let R be a relation defined on N by R={(a,b): a-b is divisible by 3}.
   a. Prove that R is an equivalence relation.
   b. Find the different equivalent classes.
2. Let f,g:R→R defined by f(x)=7x+3 and g(x)= $\frac{x-3}{7}$.
   a. Find (f o g) and (g o f).
   b. Hence find the inverse of f.
3. Let ∗ be a binary operation defined on R by a∗b= $\frac{ab}{9}$
   a. Check whether ∗ is commutative or associative.
   b. Find the identity element of ∗ .
   c. Find the inverse of the element 8.

Teacher Tips

Bell Numbers

Corresponding to every equivalence relation on a non-empty set we can define equivalence classes, which is a partition to that set. ie, number of equivalence classes on a set is equal to number of partitions of that set.

Given any set of distinct objects, how many ways can it be split into subsets? Such a division is called a **set partition**. More formally, a set partition of a set of distinct objects, S, is a set of disjoint subsets whose union is S.

For example,1. the set {a,b,c} can be partitioned in 5 ways:

{a,b,c}

One subset	
Two subsets	{a} {b,c}
	{b} {a,c}
	{c} {a,b}
Three subsets	{a} {b} {c}

The number of ways that a set can be partitioned is it's **Bell number**. Named after Eric T. Bell, the mathematician who first studied the numbers in depth

The Bell number for a set with N members, **B(N)**, is the sum of the number of ways that it can be partitioned into 1, 2, 3, ... N subsets.

Example 1. If A= { }, empty set ,then **B(0) =1**

Example 2: A={a} Partitions are {a} **B(1)=1**

Example 3: A={a, b}

Partitions are

One subset: {a, b} Two subsets: {a}, {b} **B(2) =2**

Example: 4: A={a,b,c} Partitions are

One Subset: {a,b,c}

Two Subsets: {a} {b,c},

{b},{a,c}

{c} {a,b}

Three Subsets: {a} {b} {c}

B(3)=5

Recurrence relation to find the Bell Numbers , B(n+1)= $\sum_{k=0}^{n}\binom{n}{k}$ B(n)